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REMARKS ON CHIRAL SYMMETRY BREAKING WITH MASSLESS FERMIONS

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ABSTRACT

In this talk I present recent results on Lorentz covariant correlation functions $\langle q(p_1)\overline{q}(p_2)\rangle$ on the cone $p^2 = 0$. In particular, chiral symmetry breaking terms are constructed which resemble fermionic 2–point functions of 2–D CFT up to a scalar factor.

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1. Confinement and chiral symmetry breaking are characteristic features of low energy QCD which remain puzzling from a theoretical point of view, although there has been continuing progress in the description of these phenomena during the more than 20 years since the introduction of QCD. Theoretical attempts to understand confinement of relativistic quarks and gluons from first principles center around the dual Meissner effect via monopole condensation[1,2]. Spontaneous breaking of chiral $SU(N_f)$ symmetry, on the other hand, is expected to arise as a consequence of confinement or as an instanton effect[3,4], but it is not clear which mechanism drives chiral symmetry breaking. Chiral symmetry breaking effects of confining forces have been discussed in[5,6], and it has been pointed out that in dual QCD monopole condensation not only yields confinement but also a chiral condensate through a gap equation[7]. Furthermore, Seiberg and Witten recently constructed the Wilsonian low energy effective action for $N = 2$ super–Yang–Mills coupled to hypermultiplets and determined the singularities of the quantum moduli space[8], see also[9]. They observed that in this framework flavor charged monopoles imply both confinement and chiral symmetry breaking.

In QCD we would like to understand how ordinary monopoles break chiral symmetry in spite of their chiral coupling, or whether instantons or other non–perturbative effects break chiral symmetry before confinement. This problem is also relevant for the nature of the phase transition, since the absence of an order parameter for confinement in the presence of light flavors excludes a second order phase transition, if chiral symmetry breaking is causally connected to confinement.[a] Pisarski and Wilczek pointed out that an ϵ–expansion for the corresponding σ–model indicates a first order transition for more than two light flavors, but that a second order transition in the universality class of the $O(4)$ vector model is likely to appear in case of two light flavors[11,12]. A very recent review and discussion of numerical results can be found in Ref.[13].

In this talk I report on recent results for the the correlation $\langle q(p_1)\overline{q}(p_2)\rangle$ on the orbit $p^2 = 0$, in an attempt to shed new light on the problem from an unexpected angle[14]. Dynamical breaking of chiral flavor symmetry in gauge theories is a puzzle, because it is very different from spontaneous magnetization: In a ferromagnet the interaction tends to align the dipoles, while thermal fluctuations restore disorder if the system has enough energy. Gluon exchange, on the other hand, does not necessarily align left– and right–handed fermions. Stated differently, the massless Dyson–Schwinger equation for the trace part of the fermion propagator always admits a trivial solution, if a quark condensate is not inserted *ab initio*.[b] Therefore, chiral symmetry breaking has to be implemented in unusual ways, if we want to recover it from gauge dynamics: By requiring a condensate as part of initial conditions,

[a] Without dynamical quarks the deconfinement transition is expected to be first order from triality[10].
[b] In the latter case the corresponding Dyson–Schwinger equation yields a gap equation for the condensate. The puzzle then concerns the identification of the "phonons" in QCD.

in double scaling limits, through chiral symmetry breaking boundary conditions, in chiral symmetry breaking regularizations, etc. While this does not invalidate standard approaches to the problem, it serves to remind the reader that some poorly understood mechanism leaves its footprint on the long distance properties of the QCD vacuum, and motivates the group theoretical construction of Lorentz covariant correlation functions given below.

The main ingredient of the work reported below is a mapping between massless spinors in 3+1 dimensions and primary fields, which relates the order parameter to automorphic forms under the Lorentz group. From a mathematical point of view, the novel feature of the automorphic forms under investigation is that they provide correlations between primary fields on spheres of different radii, thus providing true representations of the Lorentz group and extending the determination of correlation functions in 2D conformal field theory. The nontrivial behavior of radii under the boost sector of the Lorentz group allows for chiral symmetry preserving terms in the correlation functions which could not appear in a two–dimensional framework, while the chiral symmetry breaking terms in turn appear closely related to 2D fermionic correlation functions.

I will briefly review the evidence for dynamical chiral symmetry breaking in QCD in Sec. 2. The mapping between massless spinors in Minkowski space and primary fields on spheres in momentum space and an application to construct chiral symmetry breaking correlations will be outlined in Sec. 3.

2. There exists wide agreement that chiral symmetry breaking in QCD arises both dynamically, as a genuine QCD phenomenon, and through electroweak symmetry breaking, which in a standard scenario accounts for the quark current masses.[c] Dynamical chiral symmetry breaking is then expected to account partially for the difference between current and constituent masses[15]. From this point of view, the large discrepancy between current and constituent masses, and the fact that there is not even an approximate parity degeneracy in the hadron spectrum provides strong evidence for dynamical breaking of chiral symmetry.

Another argument in favor of dynamical breaking of chiral symmetry comes from the Gell-Mann–Oakes–Renner relation:

$$2m_q\langle\overline{q}q\rangle = -f_\pi^2 m_\pi^2 \tag{1}$$

where m_q stands for a mean value of current quark masses. This relation is expected to hold in the sense of a leading approximation in m_q, and works phenomenologically the better the smaller the value of m_q is[16,17]. While this does not strictly imply $\lim_{m_q\to 0}\langle\overline{q}q\rangle \neq 0$, it implies at least that the condensate vanishes weaker than first order in m_q.

[c] The electroweak sector also contributes to breaking of chiral $SU(N_f)$ through the axial anomaly since the charge operator Q^2 is not flavor symmetric.

The necessity of including non–vanishing condensates in QCD sum rules provides further strong indication for spontaneous breaking of chiral symmetry. This becomes particularly evident in heavy–light systems, where the condensate of the light quark is expected to contribute to the meson propagator even in the limit of vanishing current mass[18,19].

Further hints for chiral symmetry breaking are provided by 't Hooft's result that decoupling of heavy fermions does not comply with local chiral flavor symmetry[20], and indirectly through the no–go theorem of Vafa and Witten for spontaneous breaking of vector–like global symmetries in QCD[21].

Chiral symmetry breaking can also be addressed in lattice QCD with staggered fermions. In this framework non–vanishing condensates have been reported e.g. in[22,23,24,25].

This mini–review comprises a very short summary of the most compelling arguments in favor of dynamical chiral symmetry breaking. Clearly, there is insurmountable evidence that chiral symmetry breaking in QCD is not solely of electroweak origin.

3. Chiral spinors in 3+1 dimensions can be described as primary fields of conformal weight $\frac{1}{2}$ on spheres in momentum space. To exploit this observation, we work in the Weyl representation of Dirac matrices, and parametrize the unit sphere in momentum space in terms of stereographic coordinates:

$$z = \frac{p_1 + ip_2}{|\mathbf{p}| - p_3} \qquad\qquad \tilde{z} = -\frac{p_1 - ip_2}{|\mathbf{p}| + p_3} \tag{2}$$

Proper orthochronous Lorentz transformations act on these coordinates according to

$$z' = z(\mathbf{p}') = \overline{U} \circ z(\mathbf{p}) = \frac{\bar{a}z + \bar{b}}{\bar{c}z + \bar{d}} \tag{3}$$

if $E = |\mathbf{p}|$, and

$$z' = U^{-1T} \circ z(\mathbf{p}) = \frac{dz - c}{a - bz} \tag{4}$$

if $E = -|\mathbf{p}|$.

U denotes the positive chirality spin $\frac{1}{2}$ representation of the Lorentz group:

$$U(\omega) = \exp(\frac{1}{2}\omega^{\mu\nu}\sigma_{\mu\nu}) = \left(\begin{array}{cc} a & b \\ c & d \end{array} \right) \in SL(2, \mathbf{C})$$

We identify local functions written in co-ordinates $(z, \bar{z}, |\mathbf{p}|)$ and $(\tilde{z}, \bar{\tilde{z}}, |\mathbf{p}|)$ via

$$\psi(\tilde{z}, \bar{\tilde{z}}, |\mathbf{p}|) = -z\psi(z, \bar{z}, |\mathbf{p}|) \tag{5}$$

$$\phi(\tilde{z}, \bar{\tilde{z}}, |\mathbf{p}|) = \bar{z}\phi(z, \bar{z}, |\mathbf{p}|) \tag{6}$$

and these overlap conditions can be rephrased as Weyl equations:

$$(|\mathbf{p}|+\mathbf{p}\cdot\sigma)\left(\begin{array}{c}\psi(z,\bar{z},|\mathbf{p}|)\\ \psi(\tilde{z},\bar{\tilde{z}},|\mathbf{p}|)\end{array}\right)=0$$

$$(|\mathbf{p}|-\mathbf{p}\cdot\sigma)\left(\begin{array}{c}\phi(\tilde{z},\bar{\tilde{z}},|\mathbf{p}|)\\ \phi(z,\bar{z},|\mathbf{p}|)\end{array}\right)=0$$

Under (3) ϕ and ψ transform according to

$$\phi'(z',\bar{z}',|\mathbf{p}'|)=(c\bar{z}+d)\phi(z,\bar{z},|\mathbf{p}|) \tag{7}$$

$$\psi'(z',\bar{z}',|\mathbf{p}'|)=(\bar{c}z+\bar{d})\psi(z,\bar{z},|\mathbf{p}|) \tag{8}$$

if $E=|\mathbf{p}|$. Due to (5,6) this is equivalent to

$$\left(\begin{array}{c}\phi'(\tilde{z}',\bar{\tilde{z}}',|\mathbf{p}'|)\\ \phi'(z',\bar{z}',|\mathbf{p}'|)\end{array}\right)=U\cdot\left(\begin{array}{c}\phi(\tilde{z},\bar{\tilde{z}},|\mathbf{p}|)\\ \phi(z,\bar{z},|\mathbf{p}|)\end{array}\right)$$

$$\left(\begin{array}{c}\psi'(z',\bar{z}',|\mathbf{p}'|)\\ \psi'(\tilde{z}',\bar{\tilde{z}}',|\mathbf{p}'|)\end{array}\right)=U^{-1\dagger}\cdot\left(\begin{array}{c}\psi(z,\bar{z},|\mathbf{p}|)\\ \psi(\tilde{z},\bar{\tilde{z}},|\mathbf{p}|)\end{array}\right)$$

The case $E=-|\mathbf{p}|$ corresponds to $U\leftrightarrow U^{-1\dagger}$ in the equations above, but we will stick to positive energy in the sequel. Correlations for $E=-|\mathbf{p}|$ can easily be recovered from the covariance cosiderations for positive energy through a reflection $\mathbf{p}\to-\mathbf{p}$.

To make a short story even shorter, we made spin bundles look like line bundles over the orbit $p^2=0$. To take advantage of this construction, we write a spinor on the half–cone $E=|\mathbf{p}|$:

$$\Psi(p)=\left(\begin{array}{c}1\\0\end{array}\right)\otimes\left(\begin{array}{c}\bar{z}\\1\end{array}\right)\phi(z,\bar{z},|\mathbf{p}|)+\left(\begin{array}{c}0\\1\end{array}\right)\otimes\left(\begin{array}{c}1\\-z\end{array}\right)\psi(z,\bar{z},|\mathbf{p}|) \tag{9}$$

with a corresponding representation of the correlation function of massless fermions in the Dirac picture

$$\langle\Psi(p)\overline{\Psi}(p')\rangle= \tag{10}$$

$$\left(\begin{array}{cc}0&1\\0&0\end{array}\right)\otimes\left(\begin{array}{cc}\bar{z}z'&\bar{z}\\z'&1\end{array}\right)\langle\phi(\mathbf{p})\phi^+(\mathbf{p}')\rangle+\left(\begin{array}{cc}0&0\\1&0\end{array}\right)\otimes\left(\begin{array}{cc}1&-\bar{z}'\\-z&z\bar{z}'\end{array}\right)\langle\psi(\mathbf{p})\psi^+(\mathbf{p}')\rangle$$

$$+\left(\begin{array}{cc}1&0\\0&0\end{array}\right)\otimes\left(\begin{array}{cc}\bar{z}&-\bar{z}\bar{z}'\\1&-\bar{z}'\end{array}\right)\langle\phi(\mathbf{p})\psi^+(\mathbf{p}')\rangle+\left(\begin{array}{cc}0&0\\0&1\end{array}\right)\otimes\left(\begin{array}{cc}z'&1\\-zz'&-z\end{array}\right)\langle\psi(\mathbf{p})\phi^+(\mathbf{p}')\rangle$$

The 2–point functions on the right hand side transform under a factorized representation of the Lorentz group. This makes this representation very convenient for the investigation of correlations $\langle\Psi(p)\overline{\Psi}(p')\rangle$ which comply with Lorentz covariance. Stated differently, we ask which correlations of spinors of the form $\Psi(p)$ could be constructed from a non–trivial vacuum, or more general, between any Lorentz invariant states.

The investigation in Ref.[14] revealed

$$\langle\psi(\mathbf{p}_1)\psi^+(\mathbf{p}_2)\rangle = \langle\phi(\mathbf{p}_2)\phi^+(\mathbf{p}_1)\rangle = f_1\!\left(\frac{|\mathbf{p}_1|}{|\mathbf{p}_2|}\right)\frac{1+z_1\bar{z}_2}{\sqrt{|\mathbf{p}_1||\mathbf{p}_2|}}\,\delta_{z\bar{z}}(z_1-z_2) \qquad (11)$$

$$\langle\psi(\mathbf{p}_1)\phi^+(\mathbf{p}_2)\rangle = \overline{\langle\phi(\mathbf{p}_2)\psi^+(\mathbf{p}_1)\rangle} = \frac{1}{z_1-z_2}\,f_2\!\left(|\mathbf{p}_1||\mathbf{p}_2|\frac{(z_1-z_2)(\bar{z}_1-\bar{z}_2)}{(1+z_1\bar{z}_1)(1+z_2\bar{z}_2)}\right) \qquad (12)$$

where Lorentz covariance does not fix f_1 and f_2.[d] However, on dimensional grounds we infer that $f_2(x) = \frac{C}{\sqrt{x}}$, whence the correlation function is strongly peaked for parallel momenta.[e] Under the proviso that a Dirac picture makes sense in a nonperturbative problem, the orbit $p^2 = 0$ contributes to a condensate

$$tr\langle\Psi(p)\overline{\Psi}(p')\rangle = -\frac{2C}{\sqrt{|\mathbf{p}||\mathbf{p}'|}\sin(\frac{\theta}{2})} \qquad (13)$$

where θ denotes the angle between $\mathbf{p}$ and $\mathbf{p}'$.

If the correlation function in configuration space gives the positive energy contribution to a propagator of initial conditions (modulo $i\gamma_0$), as follows for the perturbative vacuum from canonical quantization, then the result above would be consistent insofar as the f_2 terms do not anticommute with γ_5, while the f_1 terms anticommute with γ_5 and imply a restriction for external momenta to be parallel. However, since we pretend to deal with a confining theory (albeit disguised in a non–perturbative vacuum), there is no reason to believe that the propagator can be reconstructed from data on a single orbit of the Lorentz group. The results above presumably make sense only within a confining theory, if observables are expressed in meson or baryon correlations. Of course, n–point functions of primary fields in 2D CFT are not unique if $n > 3$, and I expect similar ambiguities to show up in the present setting.

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[d] The correlation in the vacuum of the free theory $\langle\psi(\mathbf{p})\overline{\psi}(\mathbf{p}')\rangle = -2p\cdot\gamma|\mathbf{p}|\delta(\mathbf{p}-\mathbf{p}')$ is recovered from Eqs. (10,11,12) for $f_1(x) = \delta(x-1)$, $f_2 = 0$.

[e] Inclusion of a scale Λ allows for obvious generalizations.

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